Summary

• LifePath’s lifecycle consumption framework seeks to help individuals support a lifetime of spending from the finite amount of income they earn during their careers.

• While inflation would intuitively seem to be a concern within a lifecycle framework, there has been little research into inflation and inflation-hedging in a consumption and asset allocation context.

• This paper seeks to establish that returns for most commonly invested asset classes and income growth outpace inflation, both of which help hedge inflation, especially for young investors, and examines the opportunity cost of introducing a hedging asset class too early.

• As investors age and financial assets increase, hedging assets against inflation shocks becomes more attractive. We identify at what point to begin hedging as well as the appropriate allocation to hedging assets for older and retired investors.

• The paper describes innovations to our target date fund methodology, including the introduction of inflation regimes and the development of an inflation asset class for optimization purposes.
The lifecycle consumption framework we use to build the BlackRock LifePath® target date strategies can be seen as a working theory of how individuals earn, save and spend throughout their lifetime. The challenge we seek to solve is that individuals earn a finite amount of labor income during their working years, yet that income needs to support a lifetime of spending, including decades after the paychecks stop. We assume that they want to enjoy their labor income as fully as possible at every stage of their lives. The framework incorporates demographic, behavioral and economic insights to help individuals find a balance that allows them to enjoy their income by maximizing spending at every point in their lives while minimizing savings during their career.

On an intuitive level, inflation would seem an important consideration within a lifecycle consumption framework. After all, a 25-year old setting aside money for a retirement forty years in the future may face decades of spending power erosion based on even modest inflation. Making an explicit inflation-hedging allocation would seem to make sense. One of the questions we explore in this paper is whether that intuition is correct.

To our knowledge, this is the first research into inflation-hedging in the context of lifecycle consumption and asset allocation. In the absence of theoretically consistent, quantitative research, investment managers are left to act on heuristics, leveraging assumed inflation-hedging assets and implementing allocations without a clear articulation of the expected impact on saving and spending. There is no empirically grounded guidance as to when it may be appropriate to allocate to hedging assets, as well as how much to allocate.

In this paper, we build on our robust lifecycle consumption maximization framework and seek to quantitatively answer questions about the inflation-hedging characteristics of financial assets and human capital. We consider the opportunity cost of inflation-hedging and the point at which individuals should consider hedging inflation risk, as well as the size of the allocation they should consider. We will also describe how we have expanded our target date fund methodology to apply our findings, introduce the concept of retirement capital and offer fresh insights into human capital characteristics in relation to inflation.

At the core of our model is the concept that total wealth is comprised of both financial capital and human capital. Financial capital is an individual’s current financial wealth. Human capital is the present value of an individual’s expected future labor income. The interplay between risks and expectations of both is the primary driver of the model’s asset allocation optimization. To expand our framework and consider the demand for inflation-hedging, we need to understand the relationships between financial capital and inflation, and human capital and inflation. We begin with inflation and financial capital.
Inflation and financial capital

Selecting an inflation index. Expected inflation vs inflation shocks. Asset class returns against inflation.

What do we mean by inflation and how does our understanding of inflation shape our approach to inflation-hedging? Perhaps the most commonly cited inflation statistic is Core CPI, which is the consumer price index for all urban consumers, excluding food and energy. The Bureau of Labor Statistics randomly samples price data for a basket of goods and services, and then weights the data by their importance in the spending patterns of the population. Core CPI excludes food and energy prices on the rationale that they can be volatile, with supply disruptions and other factors that cause temporary price shocks that do not persist.

Our goal, however, is to capture the real-world spending challenges individuals face on a daily basis, and that would include food and energy costs and volatility. Therefore, we use CPI-U, which includes all prices, in the model. As with CPI, data is collected for all urban consumers, who represent approximately 88% of the population, and includes short-term workers, self-employed and unemployed people, as well as retirees and those not in the labor force. Prices are collected from approximately 4,000 housing units and 26,000 retail establishments across 87 urban areas.

When economists talk about inflation, they typically decompose it into two parts: expected and unexpected inflation. Expected inflation is just that: the normal level of inflation we’d expect to see. Unexpected or shock inflation can be loosely defined broadly as inflation above the expected level, especially if it occurs within a relatively brief period. A more rigorous approach is to use a statistical model called a Markov Switching Process to classify periods as high inflationary. The result is shown in Figure 1.

Figure 1: Identifying periods of shock inflation

Smoothed high inflation probability

CPI-U quarterly change

Figure 2 uses a technique that filters out expected inflation from total inflation, allowing us to see just how volatile shock inflation can be. We also see that in general, there appear to be “inflation regimes”, where both the level and volatility of inflation persist over some amount of time.

With expected and shock inflation in mind, there are two separate but related ways to think about inflation protection and financial assets:

**Protecting current financial capital.** This is likely how most investors think about inflation protection: hedging current financial capital against shock inflation over a relatively short time horizon. This type of hedging is often sought through allocations to assets like commodities and Treasury Inflation Protected Securities (TIPS) that may be correlated with inflation during shocks.

**Protecting future consumption.** In this instance, protection refers to reducing the uncertainty investors may have about the purchasing power of their financial capital at some point in the future. Given a sufficient investment horizon, many commonly investable financial assets, such as equities or bonds, have had historical returns that outpaced inflation, even in the presence of inflation spikes, making them a suitable hedge for future consumption.

Is there a significant difference between these types of inflation protection? Wouldn’t a 30-year old who managed to preserve financial capital during an inflation shock be in a better position for retirement than a 30-year old whose portfolio lost half its value?
Perhaps, but not necessarily. Periods of shock inflation are brief when compared to the decades-long time horizon ahead of a 30-year old investor, and if commonly invested asset class returns outpace inflation, a 30-year old may be better served remaining fully invested in growth assets. Figure 3 compares returns against inflation for a number of asset classes during each of the previous four decades. We can see that it has largely been the case that most commonly investable class returns have outpaced inflation, with the most notable exceptions in the 2000s when the Tech Bubble and the Financial Crisis led to significant losses for equities.

The outperformance of most financial assets raises the question of whether 30-year olds face an opportunity cost if they persistently hedge against shock inflation. Unless they expect to move in and out of TIPS, for example, based on inflation expectations, they would need to maintain an allocation in order to have the protection in place when it was needed. And since TIPS (as well as most frequently used inflation hedges) are likely to have lower real returns on average than the asset class it is displacing, the result may be less accumulated financial capital and lower future consumption.

We will return to opportunity cost later, but based on historical asset class performance alone, we can make a preliminary case that a 30-year old would better protect her future consumption by investing in an age-appropriate mix of diversified growth asset classes rather than a hedging asset.

Figure 3: Common asset classes against inflation, by decade

Inflation and human capital

The importance of human capital. Understanding the pattern of wage growth. Does wage growth outpace inflation?

We believe that our analysis of human capital, the second component of total wealth in our framework, differentiates our target date strategies from other providers. We define human capital as the present value of future labor earnings. Our human capital analysis and our understanding of retirement liability (the number of years and level of retirement spending) drive our decision-making process and our asset allocation.

Here are several important considerations to keep in mind about human capital:

- Human capital is finite and is (essentially) exhausted at retirement, the point at which an individual ceases earning wages.
- While human capital is often described as “bond-like” in that it provides a series of payments over time, there is considerable volatility associated with wage income that must be taken into account.
- In general, as individuals earn income, their human capital decreases (that is, their expected future income is reduced) and, ideally, their financial capital (i.e., savings) increases.
- Remaining human capital is a primary driver of the risk exposure for financial capital investment with growth exposure at its peak when human capital is at its highest.

It is also important to understand lifetime wage patterns in real terms. When individuals enter the workforce, their income reflects their education, geographic location, profession, career trajectory and dozens of other factors. As they gain experience, they are expected to develop expertise and skills that increase their value to employers and wages are expected to rise in response. Changes in income due to promotions, new skills or accreditations and so on are likely to persist and may further increase an individual’s human capital. Individual income, however, has unique risks and year-to-year volatility as injuries, family or medical leave, job losses, exceptional sales years and other factors can deliver temporary income shocks.

In order to understand the broad pattern of income growth, we research various publicly available income data sources, including the University of Michigan Panel Study of Income Dynamics (“PSID”) and individual income data from the U.S. Census Current Population Survey. There are advantages to both, with PSID providing household data across decades and the Census offering more insight on individual dynamics. For the aggregated cross sections of data shown in this paper, we use the Michigan study.

Figure 4: The Pattern of Income Wage Growth

In Figure 4, the bands around the median wage line capture the volatility of wages. The overall pattern, however, validates our hypothesis: wages rise in real terms as a function of age, at least during the first decades of a career, and peak roughly 15 years from retirement. This pattern has been historically consistent, regardless of economic conditions or where various time periods are along the inflation curve. Figure 5 gives us “snapshots” of median and mean income data by age, taken ten years apart. We can see that the distribution within each time period is largely consistent with what we’ve seen in Figure 4.

Figure 5: Average and median labor wages by age at various periods

<table>
<thead>
<tr>
<th>Year</th>
<th>Median earnings</th>
<th>Mean earnings</th>
</tr>
</thead>
<tbody>
<tr>
<td>1985</td>
<td>$80K</td>
<td>$60K</td>
</tr>
<tr>
<td>1995</td>
<td>$80K</td>
<td>$60K</td>
</tr>
<tr>
<td>2005</td>
<td>$80K</td>
<td>$60K</td>
</tr>
<tr>
<td>2013</td>
<td>$80K</td>
<td>$60K</td>
</tr>
</tbody>
</table>


Having established the pattern that lifetime wages are expected to follow, we turn to the relationship between inflation and human capital. Let’s consider two possibilities:

**Income growth can be shown to outpace inflation:**
If this is true, human capital may hedge inflation on its own, diminishing the need for allocating to an inflation-hedging asset class within a target date fund.

**Income growth fails to outpace inflation, or does so unreliably:** In this case, human capital may be an ineffective hedge and greater consideration may be warranted for hedging financial capital asset class within the glidepath.
There are a number of economic theories that suggest wages eventually rise in response to price increases, and therefore keep pace with inflation. We sought an empirical answer, however, based on historical income data. Figure 6 compares overall wage growth (regardless of age) against inflation for four decades beginning with the 1970s. The charts show wages over a ten-year period for various wage cohorts from the top 1 percent of income earners through the lower wage earners. We see that across all the periods considered, every wage cohort beat inflation. It may be particularly noteworthy that this held true even during the high inflation witnessed in the 1970s.

Figure 6: Comparing wages to inflation by decade

More recently, the issue of wage stagnation has gotten attention,¹ which raises the question of whether the relationship between inflation and human capital will continue. Thus far this decade, wage growth has been stronger for higher income brackets, while median and the lowest income quartile has been relatively flat.² Inflation has been modest by historical standards as well, and lately there has been some potential upward pressure for lower quartile wage earners as some states and municipalities have increased minimum wage requirements. We will continue to research the relationship, but the historical observed data suggests that wage growth will continue to outpace inflation. The decade ending in 2010, it should be noted, had modest wage growth compared to prior periods, but nonetheless outpaced inflation.

Finally, the income curve in Figure 4 suggests that wage growth for younger cohorts should outpace inflation more strongly than older cohorts. In Figure 7, we look at wage growth by age cohort for each of the last four decades. As expected, the youngest cohort showed the strongest growth against inflation. Interestingly, most cohorts, including the oldest, also generally outpaced inflation, with the one notable exception being the 55+ cohort in the 1980s.

At this point, we can state that overall wage growth for younger cohorts outpaced inflation, and at least tracked inflation for all cohorts, including those within ten years of the traditional retirement age of 65.

This suggests that human capital is a suitable hedge of short-to-medium term purchasing power, while, as shown in Figure 3, long-term future consumption is supported by investment in assets, in particular equities, with returns that have been shown to outpace inflation over longer periods. Younger investors therefore already hold sufficient inflation-hedging assets (including human capital and equities) meaning very little in the way of additional hedging assets may be necessary.

Next, we turn to the relationship between financial and human capital, and the objective of supporting future consumption, by proposing the concept of retirement capital.
Utility-driven model. The opportunity cost of inflation-hedging.

As we stated earlier, the goal of our consumption maximization model is to help individuals maximize consumption throughout their lives while minimizing the need to save. To attempt to accomplish this, we need to understand consumption both during working years and in retirement. Figure 8 plots median income data alongside optimal consumption, as determined by our lifetime consumption model. We use a utility factor to determine the most efficient consumption across the lifecycle. (It is worth noting we validate our consumption projections against empirical spending data and have found them to be consistent with our expectations.) The chart gives us several key insights.

The chart helps us understand the optimal savings rate by age. During an individual’s career, her consumption is supported by current wages until a point near retirement, where she begins to spend her savings. Early in her career, nearly all her wages are needed to support consumption, allowing very little saving. Wages increase more quickly than spending, however, allowing saving to increase as well, peaking in the late middle portion of a career.

By extending the consumption line beyond the retirement date (here assumed to be 65) we see the retirement outcome we hope to achieve. If our consumption maximization objective is successful, consumption (or as an individual might understand it, spending) continues in the smoothed arc seen here. To accomplish this, our model seeks to solve for the optimal asset allocation for an increasing inflow of financial capital, offset by a diminishing store of human capital to support our projected level of retirement consumption. To help understand the interplay of the various elements, we introduce the idea of retirement capital.

Figure 8: Labor income and spending

Data: University of Michigan Panel Study of Income Dynamics, BlackRock, January 2019. For more on consumption utility estimates, see the appendix.
Retirement capital is the projected value of current financial capital at age 66, taken here as the first year of retirement. Retirement capital projections reflect assumptions about savings, as well as the risk and return assumptions for the underlying asset allocation. As Figure 9 shows, total capital (the sum of human and retirement capital) decreases over time as investors deplete their human capital and the portion of retirement capital increases. At the point of retirement, all consumption will be funded out of retirement capital.

At the beginning of a career, inflation may be more approximately hedged by human capital and investment in growth assets. At the end of a career, total wealth is comprised almost entirely by retirement capital.

Human capital and its inherent inflation hedge is gone. In LifePath, as is the case with most target date and lifecycle strategies, investment in growth assets has been reduced. Even if the retirement capital investment mix is still expected to outpace inflation, an inflation shock and downside volatility may significantly affect a retired investor’s consumption. At some point, it makes sense to begin hedging against shock inflation.

But hedging against inflation shocks, as we stated, may entail an opportunity cost – that is, giving up some expected return by replacing the potential growth of an asset class with the lower expected returns of hedging assets. Let’s put the opportunity cost in context.
Many commonly used inflation-hedging asset classes, such as TIPS, which we will use in this example, have low, but consistent, real return expectations over time. An allocation to a hedging asset therefore comes at the expense of the higher expected real returns of a growth asset (and its associated higher volatility). Lower expected returns require an adjustment in either saving or spending behavior.

In Figure 10, we compare the lifetime consumption projection for a default glidepath (based on BlackRock’s LifePath strategy) to a glidepath with a persistent allocation to TIPS. To offset some of the lower expected returns for the Equity-TIPS glidepath, we increased the equity allocation marginally. Despite the increased equity allocation, lower expected return for the Equity-TIPS glidepath requires additional saving (and therefore less spending) during the working years. Even so, the projection is for lower spending in retirement, with about 10% less consumption across the lifecycle.

Inflation hedging comes at a cost; the question is, at what age does paying that cost start to make sense? Incorporated into our utility framework is the assumption that as investors age and approach retirement, they begin to value certainty about their retirement capital over the marginal benefit of incremental increases in retirement spending ability. They become increasingly willing to make the trade-off in terms of bearing the opportunity cost. We turn to the question of when to begin hedging inflation shocks and what is the appropriate size of the allocation.

**Figure 10: Persistent inflation hedging may reduce lifetime spending**

For illustration only. Source: BlackRock, January 2019. For more on consumption utility estimates, see the appendix.
Solving for inflation

Expanding the lifecycle model. When to begin hedging.

Our lifecycle model provides us with a single, unified framework for making investment, savings and spending decisions. Our utility of consumption framework quantitatively solves for optimal consumption, savings and investment decisions. It is this framework that will help us move from more qualitative statements like “investors should be more concerned about inflation as they approach retirement” to a precise allocation to inflation hedging assets as a function of age.

To appropriately include inflation in our framework, we need to consider:

The behavior of financial assets in relation to inflation:
In Figure 2, we isolated periods of “shock” inflation. While the likelihood of sudden periods of high inflation may be low, they are potentially devastating for the investors with significant amounts of retirement capital at risk. Critically, shock inflation has tended to occur when many financial assets have performed poorly or shown increased volatility.

Defining inflation hedging properties: In the context of shock inflation, hedging refers to asset class movements in response to changes in inflation — in other words, their correlation. Figure 11 shows the correlation of various asset classes to overall inflation, followed by a breakout of their correlation in expected inflation regimes and unexpected, or shock, inflation. These relationships will help shape our expectations as to how potential hedging and other asset classes may behave in inflation shocks.

Figure 11: Correlations of asset classes with inflation


<table>
<thead>
<tr>
<th>Asset class</th>
<th>Index name</th>
<th>Start date</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cash</td>
<td>ICE BofAML 3 Month Treasury Bill Index</td>
<td>12/30/77</td>
</tr>
<tr>
<td>Real Estate</td>
<td>FTSE EPRA/NAREIT Developed Index Net TRI USD</td>
<td>2/28/05</td>
</tr>
<tr>
<td>Commodities</td>
<td>Bloomberg Commodity Index</td>
<td>1/31/91</td>
</tr>
<tr>
<td>TIPS</td>
<td>Bloomberg Barclays US Govt Inflation Linked Bonds Total Return Index</td>
<td>2/28/97</td>
</tr>
<tr>
<td>Int’l Equity</td>
<td>MSCI ACWI ex USA IMI Net Total Return USD Index</td>
<td>1/31/95</td>
</tr>
<tr>
<td>U.S. Large Cap</td>
<td>Russell 1000 Index</td>
<td>12/29/78</td>
</tr>
<tr>
<td>U.S. Small Cap</td>
<td>Russell 2000 Index</td>
<td>12/29/78</td>
</tr>
<tr>
<td>U.S. Bonds</td>
<td>Bloomberg Barclays US Agg Total Return Value Unhedged USD</td>
<td>1/30/76</td>
</tr>
</tbody>
</table>
Next, we expanded our framework, by incorporating two innovations into the asset allocation optimization process:

**Inflation proxy:** Within our research model, we use an *Inflation Asset*, which is an index designed to mimic inflation hedging. This index is a research tool designed to help us understand the optimal asset allocation. It is not investable, but in practical application of our findings we seek to recreate its behavior using various combinations of TIPS, commodities, real estate investment trusts (REITs) and other hedging assets depending on our research.

**Inflation state variables:** At the risk of over-simplifying, state variables are used to capture the current “state” of a dynamic system so that we can make statistical predictions about future outcomes. Our lifecycle model currently solves for the appropriate equity allocation using two state variables, age and financial capital. In order to account for the two types of inflation we have identified, the behavior of financial assets within each, and the potential impact on consumption, we have introduced a third “state variable” to our lifecycle model. (See the Appendix for a fuller explanation.)

Using the expanded framework, we run simulations to estimate returns and consumption outcomes for combinations of equities, fixed income and the inflation asset in different inflation regimes to determine the optimal asset allocation. The results are shown in Figure 12, which gives the familiar glidepath, along with the optimal allocation to the inflation asset as a function of age.

![Figure 12: Inflation factor glidepath](image)

For illustration only. Based on BlackRock data.
At a high level, the results can be summarized by two key points:

First, as discussed, there is little benefit to increasing the certainty of the consumption outcome in exchange for lower returns when investors are young. However, in a high inflationary regime, where the volatility of both the equity and fixed income assets is much higher, the appropriateness of inflation-hedging asset increases. Since shock inflation periods are, by definition, difficult to predict, once an allocation to an inflation-hedging asset is deemed necessary, it needs to be persistent from that point forward.

Second, the appropriateness of including an inflation-hedging asset class increases with age and the accumulation of financial capital. Hedging creates greater certainty around expected consumption, which is consistent with our consumption maximization objective, and with our behavioral financial assumption that older investors value higher certainty of outcomes over incremental increases in consumption that may come with higher volatility.

Our optimization, based on various inflation regimes seen in the United States in the past decades and the impact it has on asset returns, shows that individuals should potentially start allocating to an inflation-hedging asset class around age 50 and should be comfortably protected in retirement with an approximately 15% portfolio allocation. (For more on our optimization and the lifecycle model, please see the Appendix.)

Figure 13 returns to the comparison made in Figure 10, but it includes a third result, illustrating lifetime consumption for the delayed hedging glidepath. By delaying allocation to a hedging asset class until around age 50, and thereby allowing fuller investment in growth assets until then, consumption during an individual’s working years is minimally affected. Once in retirement, consumption is marginally lower, but it is much less impacted than in the persistently hedged glidepath. We believe the result meets our objective of supporting greater consumption across the lifecycle and, most critically, greater certainty of maintaining consumption consistently across the lifecycle.

Figure 13: Consumption with delayed hedging

For illustration only. Based on BlackRock data.
Conclusion: Delivering the retirement goal

The goal of any target date fund is to provide a source of retirement spending. In the absence of any other guiding principles or objectives, a target date fund manager is free to make a number of intuitive assumptions, including the need for investment returns to augment our savings, the need to curb investment risk as retirement approaches and, potentially, the need to protect retirement capital against inflation shocks.

Our consumption maximization objective, however, requires more rigor, insight and data. Fortunately, we don't have to rely on intuition. Our lifecycle model and consumption utility framework are a powerful tool that incorporates a holistic understanding of income, saving, investing and longevity, including their attendant risks and variability. Nor do we have to rely on intuition when it comes to the question of making an explicit inflation-hedging allocation.

To the best of our knowledge, this is the first study to state, based on quantitative analysis, when to begin inflation-hedging and how much to allocate to the hedging asset class. Our conclusions are based on first-principle analysis as well as our comprehensive lifecycle modeling of the problem. We have found:

- Since wages tend to outpace inflation, especially for all but the oldest cohort of workers, human capital inherently hedges inflation.
- Given a sufficiently long investment horizon, most asset class returns will eventually overcome the downside volatility that accompanies inflation shocks.
- Therefore, the demand for short-term inflation-hedging for individuals in early- to mid-career is quite low.
- Then at approximately 50 years old, as the transition from spending from wages to spending from savings nears, the demand for inflation hedging begins.
- And finally, that an allocation of up to 15% to an inflation-hedging asset class provides an appropriate tradeoff between growth and increased certainty of consumption.

We believe that this expansion of our methodology to deliver quantitative guidance to the question of inflation and inflation-hedging is consistent with the longstanding objectives of the BlackRock LifePath target date fund strategies. We believe our findings have implications for target date allocations across the glidepath.
Appendix: Evolving the lifecycle consumption framework

Our lifecycle model was described in detail in our previous publications Reexamining Too versus Through, New Research into and Old Debate (May 2014) and Inflation Demand in Lifecycle Investing (February 2019). This appendix describes the optimization that is the core of the framework and how it was expanded to account for inflation.

The objective of our lifecycle framework is to determine optimal savings rates and optimal investment allocations so that investors maximize consumption throughout their lifetime, subject to a reasonable range of outcomes.

In order to do this, the framework needs to account for how individuals earn income and their capacity for saving, the risk and uncertainties around labor income, capital market assumptions, behavioral preferences surrounding risk, and probabilistic mortality estimates which form levels (and length) of consumption in retirement. It’s built around the following optimization.

\[ V_{i,t}(X_{i,t}) = \left\{ (1 - \delta)C_{i,t}^{1-\gamma} + \delta E_t \left[ p_t \left( V_{i,t+1}(X_{i,t+1}) \right)^{1-\gamma} + b(1 - p_t)X_{i,t+1}^{1-\gamma} \right] \right\} ^{1+\delta} \]  

(1)

where:

\[ X_{i,t+1} = Y_{i,t+1} + (X_{i,t} - C_{i,t})(1 - \alpha_{i,t})r_{eq,t+1} + (1 - \alpha_{i,t})r_{b,t+1} \]  

(2)

The table below describes the variables.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>(X_{i,t})</td>
<td>Wealth/Cash-on-hand in period (t)</td>
</tr>
<tr>
<td>(C_{i,t})</td>
<td>Consumption in period (t)</td>
</tr>
<tr>
<td>(\alpha_{i,t})</td>
<td>Equity allocation in period (t)</td>
</tr>
<tr>
<td>(Y_{i,t+1})</td>
<td>Income in period (t+1)</td>
</tr>
<tr>
<td>(r_{eq,t+1})</td>
<td>Equity return between period (t) and (t+1)</td>
</tr>
<tr>
<td>(r_{b,t+1})</td>
<td>Bond return between period (t) and (t+1)</td>
</tr>
</tbody>
</table>

The following breakdown of Equation (1) may help explain key points:

\[ V_{i,t}(X_{i,t}) = \left\{ (1 - \delta)C_{i,t}^{1-\gamma} + \delta E_t \left[ p_t \left( V_{i,t+1}(X_{i,t+1}) \right)^{1-\gamma} + b(1 - p_t)X_{i,t+1}^{1-\gamma} \right] \right\} ^{1+\delta} \]

**Callout 1:** This is the utility of an individual’s current state, which is a function of current financial wealth, indicated by \(X_{i,t}\). Utility can be thought of as a measure of happiness, and the more financial wealth an investor has, the happier they are.

**Callout 2:** This is current spending. At the risk of oversimplifying, it’s understood that individuals derive happiness by spending their money. There is a trade-off, however, between spending money now and being happy now and spending money later and being happy “later”. Individuals are generally happiest if they can spend consistently across time.

**Callout 3:** This is the happiness an individual derives from having money to spend in the future, and, includes the probability of surviving from one period to the next (\(p_t\)).

These three portions of the optimization capture the balance that our consumption maximization framework seeks to achieve. At each point in time the investor has to decide how much to consume and how to allocate wealth among stocks and bonds. The wealth variable \(X_{i,t}\) is related to consumption through Equation (2) which is often referred to as the law of motion.

The main innovations to the model are including an inflation proxy as a third asset, and the addition of a state variable which captures inflation regimes. These changes result in the new optimization below.

\[ V_{i,t}(X_{i,t}, R_t) = \left\{ (1 - \delta)C_{i,t}^{1-\gamma} + \delta \sum_{k \in \{1, 2\}} E_t \left[ p_t \left( V_{i,t+1}(X_{i,k,t+1}, R_k) \right)^{1-\gamma} + b(1 - p_t)X_{i,t+1}^{1-\gamma} \right] \right\} ^{1+\delta} \]  

(3)

and:

\[ X_{i,k,t+1} = Y_{i,t+1} + (X_{i,t} - C_{i,t})(\alpha_{i,t}r_{eq,t}^{\text{nom}} + \tau_{i,t}r_{k,\text{inf},t}^{\text{nom}}) + (1 - \alpha_{i,t} - \tau_{i,t})r_{b,t}^{\text{nom}} - r_{k,\pi,t} \]  

(4)

The variables in these equations are in the table on the next page.
Variable | Description
---|---
$X_{i,t}$ | Wealth/Cash-on-hand in period $t$
$R_i$ | Inflation regime
$C_{i,t}$ | Consumption in period $t$
$\alpha_{i,t}$ | Equity allocation in period $t$
$\tau_{i,t}$ | Allocation to inflation asset
$Y_{i,t+1}$ | Income in period $t+1$
$r^{\text{nom}}_{k,\text{eq},t}$ | Equity return from $t$ and $t+1$ (conditional on Regime $k$)
$r^{\text{nom}}_{k,b,t}$ | Bond return from $t$ and $t+1$ (conditional on Regime $k$)
$r^{\text{nom}}_{k,\text{in},t}$ | Real asset return from $t$ and $t+1$ (conditional on Regime $k$)
$r^{\text{nom}}_{k,v,t}$ | Inflation between periods $t$ and $t+1$
$p_i$ | Probability of surviving from period $t$ to $t+1$
$\gamma$ | Bequest motive
$\psi$ | Risk aversion
$\nu$ | Elasticity of intertemporal substitution
$\delta$ | Discount factor
$p_{j,k}$ | Probability of moving to Regime $k$, conditional on being in Regime $j$

The callouts highlight a change from the previous problem, and are described in more detail below:

- **Blue callout:** This is the inflation regime. In our new specification, utility is now a function of both wealth and the current inflation regime.
- **Green callout:** This is the probability of moving to Regime $k$, conditional on being in Regime $j$. Through this term we capture the persistence of inflation (persistence means that inflation in two periods is not independent, or that inflation tomorrow depends on inflation today).
- **Purple callout:** This is the weight and return of the third inflation factor asset: $\tau_{i,t}$ is the weight, and $r^{\text{nom}}_{k,\text{in},t}$ is the return. Note the $k$ subscript, which means that the return is conditional on being in Regime $k$.
- **Orange callout:** This is the return of inflation. Again, this return is conditional on being in Regime $k$.

The optimization problem then is to find $C_{i,t}$, $\alpha_{i,t}$, and $\tau_{i,t}$ so that utility is maximized for a given level of wealth. Equity $r^{\text{nom}}_{k,\text{eq},t}$ and bond $r^{\text{nom}}_{k,b,t}$ returns are conditional on the inflation regime. We use backward induction to solve the problem.

A simplified way of understanding the innovations added to our lifecycle model to account for inflation can be seen here:

**A lifetime consumption model with inflation**

### 1. Current Age
- **Wealth**
- **Inflation regime**
- **Savings/Wealth**
- **Equity allocation**
- **Real asset allocation**

### 2. Realized Returns & Assumptions
- **Consumption**
- **Realized labor income**
- **Remaining human capital**
- **Equity/fixed income returns**
- **Survival**
- **Inflation realization**
- **Real asset return**

### 3. Current Age +1
- **Wealth**
- **Inflation regime**
- **Savings/Wealth**
- **Equity allocation**
- **Real asset allocation**

1. Every year, the model takes into account current age, current financial wealth and inflation regime, and makes decisions about consuming vs saving, and how to allocate financial assets.
2. Each year, investment returns and labor income are realized and added to the financial wealth for the next year, while human capital assumptions drive asset allocation decisions and expectations about future income and future savings.
3. Each year, the process is repeated based on current financial wealth and age. After retirement, labor income falls out of the model.
Vincent Cocula, PhD, Director, is a member of BlackRock’s Multi-Asset Strategies LifePath group. Vincent is responsible for Research and Development for the LifePath franchise globally. He earned a PhD in Chemistry from University of California at Los Angeles.

Fiona Sloof, Associate Researcher, member of Multi-Asset Strategy LifePath Group. Fiona is responsible for research and development for LifePath. She graduated with an MPhil in Economics from the University of Oxford in 2016 where she specialized in econometric modeling.

Matthew O’Hara, Ph.D., CFA, Managing Director, is the co-head of LifePath. As part of the Multi-Asset Strategies (MAS) group, Dr. O’Hara is responsible for all investing aspects of lifetime asset allocation globally. Dr. O’Hara has been a lecturer in the MFE program at UC Berkeley since 2012 and serves on the board of the CFA Society of San Francisco.

The authors would like to thank Andrew Ang, PhD, Managing Director, Factor-Based Strategies Group for his contributions to this research.

Editor: Tony Mastrogiorgio, Director, U.S. Defined Contribution Marketing

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